Best distribution and plotting positions of daily maximum flood estimation at Ona River in Ogun-Oshun river basin, Nigeria

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Abstract: The paper discusses how Normal, Lognormal, and log-Pearson type 3 distributions were investigated as distributions for modelling at-site annual maximum flood flows using the Hazen, Weibull, and California plotting positions at Ogun-Oshun river basin in Nigeria. All the probability distributions when matched with Weibull plotting position gave similar values near the center of the distribution but varied considerably in the tails. The Weibull plotting position when matched with Normal, Log-normal and Log Pearson Type III probability distributions gave the highest coefficient of determinations of 0.967, 0.987, and 0.986 respectively. Hazen plotting position gave minimal errors with the RMSE of 6.988, 6.390, and 6.011 for Normal, Log-normal, and Log-Pearson Type III probability distributions respectively. This implies that, predicting statistically using Hazen plotting position, the central tendency of predicted values to deviate from observed flows will be minimal for the period under consideration. Minimum absolute differences of 2.3516 and 0.5763 at 25- and 50-year return periods were obtained under the Log-Pearson Type III distribution when matched with Weibull plotting position, while an absolute difference of 0.2338 at 100-year return period was obtained under the Log-Pearson Type III distribution when matched with California plotting position. Comparing the probability distributions, Log-Pearson Type III distribution with the least absolute differences for all the plotting positions is the best distribution among the three for Ona River under Ogun-osun river basin study location.

Keywords: probability distributions, plotting positions, maximum flood, estimation, statistical modelling, Nigeria


1 Introduction

Floods are natural hazards causing loss of life, injury, damage to agricultural lands, and major property losses (Fill and Stedinger, 1995). One method of decreasing flood damages and economic losses is to use flood frequency analysis for determining efficient designs of hydraulic structures. In hydrology, estimation of peak discharges for design purposes on catchments with only limited available data has been a continuing problem (Blazkovaa and Bevenb, 1997). A promising and elegant approach to this problem is the derived flood frequency curve. Reliable estimates of flow statistics such as mean annual flow and flood quantiles are needed, however, historical data that are needed to estimate these statistics are not always available at the site of interest or available data may not be representative of the basin flow because of the changes in the watershed characteristics, such as urbanization (Pandey and Nguyen, 1999; Ouarda, et al, 2006).

In practice, design floods are often estimated on the basis of a single site and/or regional flood-frequency analysis (Burn, 1990). An optimum design can be achieved with proper flood frequency and risk analyses (Saf, 2008). However, design floods estimated by fitted distributions are prone to modelling and sampling errors (Alila and Mtiraoui, 2002). Several researchers have

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investigated different distributions for application to flood-frequency analysis (Cunnane, 1989; GREHYS, 1996; Blazkova and Beven, 1997; Saf, 2008).

The available historical hydrometric data especially in developing countries can be short, limited or nonexistent (Fill and Stedinger, 1995) to the extent that it is far from being representative of the region under consideration, or getting it may be expensive, difficult, or time consuming (Öztekin, Karaman and Brown, 2007; Patel, 2007). Most frequent uses of statistics in hydrology all over the world have been that of frequency analysis, which were largely in the area of flood flow estimation. Best probability distributions that can be used in various situations are based on certain properties of such distributions (Haan, 1994). Hydrologist finds it difficult to make accurate prediction of flood estimates using limited historic information of runoff, rainfall, river stages. These can be attributed to lack of trained personnel and equipment for adequate assessment of these quantities on systematic basis in Nigeria (Adeboye and Alatise, 2007).

The distributions suggested for fitting flood extremes data have been many (Singh and Strupczewski, 2002). Öztekin, Karaman and Brown (2007) applied parameter estimation methods to a comprehensive list of different distributions. Different studies were undertaken on distribution selection for flood data all over the world. The three-parameter log-Pearson type 3 distribution is the most frequently used distribution in the USA, whereas the generalized extreme value distribution in Great Britain, the lognormal distribution in China (Singh and Strupczewski, 2002). Several flood distributions have also been studied, for example in USA (Wallis, 1988; Vogel, Thomas and McMahon, 1993); UK, Australia, Italy Scotland, Turkey and Kenya (Haktanir, 1991; Mutua, 1994; Abdul Karim and Chowdhury, 1995). There is no question that hydro-climatic regimes may be different for different regions, but the differences in regimes should serve as a hydro-physical basis for choosing a particular distribution. Therefore, selection of an appropriate distribution needs closer attention.

Probability distribution functions of continuous random variables are used to fit distributions in hydrology. All plotting position relationships give similar values near the center of the distribution but may vary considerably in the tails (Hann, 1994). Several plotting position relationships are presented in Chow (1964) while Haan (1994) suggested the use of California, Hazen and Weibull plotting position relationships as the three commonly used relationships positions satisfying the Gumbel (1958) five criteria for plotting position relationships. Similarly, Abida and Ellouze (2007) opined that the most commonly applied distributions now being the Gumbel (EV1), the Generalized Extreme Value (GEV), the Log Pearson Type III (LP3), and the Three parameter Lognormal (LN3).

This work applied three commonly used distributions and three different plotting position relationships (Table 1) to select the best flood frequency distribution that best fits the annual maximum flood flows of Ona River under Ogun-oshun river basin development in Nigeria.

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<th>Name</th>
<th>Source</th>
<th>Relationship</th>
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</tr>
<tr>
<td>Hazen</td>
<td>Hazen(1930)</td>
<td>( \frac{2m-1}{2n} )</td>
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<td>Weibull</td>
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2 Materials and methods

2.1 Study area

Ogun-Osun River Basin Development Authority (OORBDA) which is a parastatal of the Federal Ministry of Water Resources and Rural Development. OORBDA area covers the whole of Osun, Oyo, Ogun and Lagos States and has an estimated land area of 66,264 km². It is drained by two main rivers – Ogun and Osun Rivers (after which it is named) and a number of tributaries and smaller rivers namely; Sasa, Ona, Ibu, Ofiki and Yewa rivers. OORBDA lies between latitudes 6°30’ - 8°20’ N and longitudes 3°23’ - 5°10’ E (Figure 1). The data on mean annual rainfall for 30 years over OORBDA show a
variation from about 1,150 mm in the northern part to around 2,285 mm in the southern extremity. The monthly rainfall distribution shows a distinct dry season extending from November through March and a rainy season divided into two periods: April – July and September – October.

Adapted from: Joshua and Oyebanjo (2009)

Figure 1  Ogun-Oshun River Basin development authority (OORBDA) coordinates

Ibadan city has a humid equatorial climate with warm temperatures, high humidity and rainfall (Eze, 1997). International Institute of Tropical Agriculture (IITA) meteorological station is about 500 m northwest of Ona River at 7°29' N and 3°54' E (Figure 1) which is a tributary of Osun River. Annual rainfall for Ibadan is between 1,000-1,600 mm, with the mean around 1,270 mm (Lal, 1993). Approximately, 50% of the average annual rainfall occurs between April and July while 40% occurs between August and October. November to March is usually the driest months and temperatures tend to be higher. Mean day length of this latitude is 12 hours, ranging from a minimum of 11.5 in December to a maximum of 12.7 hours in June.

In determining the best distributions for daily flood estimation for Ona River at Ogun-Oshun River Basin, three plotting positions and three probability distributions were plotted against the annual maximum discharges of the river. Furthermore, the coefficients of determination, root mean square errors (RMSE) and absolute differences between predicted and observed discharges were used to determine how well the predicted discharges were able to predict the observed discharges.

2.2 Flood frequency analysis methodology

(i) Normal distribution

\[ f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \]

\[-\infty < z < \infty \quad -\infty < z < \infty. \]  \hspace{1cm} (1)

where, \( z \) is standard normal variable; \( e \) is exponential.

The statistical parameters are;

\[ \text{Mean} = \bar{Q} = \frac{1}{n} \sum_{i=1}^{n} Q_{\text{max}} \]  \hspace{1cm} (2)
The frequency factor or standard normal variable, $z$, can be approximated by the empirical relation (David A. Chin 2006, P319).

\[ z = w - \frac{2.515517 + 0.80285w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3} \]  

where, \( w = \ln \left( \frac{1}{P} \right) \), \( 0 < P \leq 0.5 \)  

$P$ is the probability of exceedance, where $P>0.5$, $1-P$ is substituted for $P$. The value of $z$ computed using the equation above is given a negative sign; the error in using the equation to estimate the frequency factor is less than 0.00045 (Chin, 2006).

The predicted floods at various return periods were determined using the mathematical expression:

\[ Q_T = \bar{Q} + z\cdot s_Q \]  

where, $Q_T$ is variate of the annual maximum discharges at return period $T$ (years); $\bar{y}$ is the logarithm mean of annual maximum discharges, m$^3$/s; and $s_y$, $s_Q$, $z$ are as previously defined.

The statistical variate and the predicted discharges are:

\[ y_T = \bar{y} + z\cdot s_y \]  

\[ Q_T = 10^{y_T} \]  

where, $y_T$, $y_T$ is variate of the annual maximum discharges at return period $T$ (years); $\bar{y}$ is the logarithm mean of annual maximum discharges, m$^3$/s; and $Q_T$, $z$, $s_y$ are as previously defined.

(iii) Log Pearson Type III distribution: The pdf of this distribution is given as:

\[ f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} (x - \gamma)^{\alpha-1} e^{-(x-\gamma)^\beta}, \ x \geq \gamma \]  

The mean, variance and skewness coefficient of the three parameters ($\alpha$, $\beta$ and $\gamma$) gamma distribution is given by:

\[ \mu = \gamma + \alpha \beta, \ \sigma^2 = \alpha \gamma, \ g_3 = \frac{2}{\sqrt{\alpha}} \]

The random variable is first transformed using the relation,

\[ Y = \ln X \]

The Pearson Type III distribution is also called the Three-Parameter Gamma distribution, the frequency factor depend on both the return period, $T$, and the skewness coefficient, $g_3$. If the skewness coefficient falls between -1 and +1, approximate values of the frequency factor for the Gamma/Pearson Type III
distribution, \( K_T \), can be estimated using the relation (Chin, 2006).

\[
K_T = \frac{1}{3k} \left\{\left[\hat{x}_T - k\right] + 1\right\} - 1
\]

(15)

where, \( \hat{x}_T = \bar{x} \) (previously defined) is the standard normal variate corresponding to the return period, \( T \) and \( k \) is related to the skewness coefficient by:

\[
k = \frac{g_x}{6}
\]

(16)

When the skewness, \( g_x \), is equal to zero, then \( K_T = \hat{x}_T \) and the Log Pearson Type III distribution is identical to the Lognormal distribution (Chin, 2006).

The coefficient of skewness \( g_x \) (Haan, 1994) is given as:

\[
g_x = \frac{1}{(n-1)(n-2)} \sum_{i=2}^{n} (y_i - \bar{y})^3
\]

(17)

The value of \( Y \) with return period \( T \), \( y_T \) is given by:

\[y_T = \mu_y + K_T \cdot s_y \]

(18)

Where \( \mu_y \) and \( s_y \) are as previously defined; \( K_T \) is the frequency factor (\( z \)) of the Pearson Type III distribution with return period \( T \). The value of the original variable, \( x \), with the return period \( T \), \( x_T \), is then given by:

\[x_T = \ln y_T^{-1} = e^{\mu_y}
\]

(19)

### 3 Results

The time series plot of maximum or peak annual flows and mean of the 18 years annual flows is as shown in the annual hydrograph Figure 2. The highest discharge of 271 m\(^3\)/s was observed in 1992 and declined to 178.6 m\(^3\)/s in 1994. The minimum peak flow for Ona River in the Ogun-Oshun river basin was 121.2 m\(^3\)/s in the year 1983. Minimum and maximum mean annual flows were 79.2 and 145.6 m\(^3\)/s for 1983 and 1992 respectively. The difference in flow magnitudes can be attributed to intermittent ephemeral nature of the stream flow which usually dry up or have reduced flow in the peak dry season or years with reduced rainfall events resulting in droughts.

![Figure 2 Annual hydrograph of Ona River](image-url)

(i) Normal distribution

The regression equation for predicted values of Normal distribution with Hazen plotting position is given by \( y = 124.1e^{0.788x} \) (\( R^2 = 0.943 \)), Weibull plotting position gives \( y = 128.1e^{0.739x} \) (\( R^2 = 0.967 \)) while California plotting position is \( y = 126.7e^{0.772x} \) (\( R^2 = 0.964 \)). The Root Mean Square Error (RMSE) for Hazen, Weibull and California plotting positions are 6.988, 7.870 and 9.979 respectively. The return periods for 25, 50 and 100 year period with Hazen plotting position are 191.41, 190.53 and 190.08 with absolute differences of 67.4247, 71.1219 and 72.9706 respectively. The return periods for 25, 50 and 100 year period with Weibull plotting position are 191.42, 190.53 and 190.09 with absolute differences of 72.0310,
74.0166 and 75.0094 respectively. Also, the return periods for 25, 50 and 100 year period with California plotting position are 191.41, 191.41 and 190.09 with absolute differences of 72.3402, 75.0567 and 75.0906 respectively.

(ii) Log-normal distribution

The regression equation for predicted values of Log-Normal distribution with Hazen plotting position is given by \( y = 125.5e^{0.766x} (R^2 = 0.977) \), Weibull plotting position gives \( y=128.0e^{0.728x} (R^2=0.987) \) while California plotting position is \( y = 124.9e^{0.800x} (R^2 = 0.948) \). The Root Mean Square Error (RMSE) for Hazen, Weibull and California plotting positions are 6.390, 7.408 and 16.618 respectively. The return periods for 25, 50 and 100 year period with Hazen plotting position are 120.82, 115.15 and 112.31 with absolute differences of 3.1595, 4.2586 and 4.8082 respectively. The return periods for 25, 50 and 100 year period with Weibull plotting position are 123.11, 118.75 and 116.58 with absolute differences of 3.7233, 2.2364 and 1.4930 respectively. Also, the return periods for 25, 50 and 100 year period with California plotting position are 123.36, 119.15 and 117.05 with absolute differences of 4.2930, 2.7969 and 2.0488 respectively.

(iii) Log-Pearson Type III

The regression equation for predicted values of Log-Pearson Type III distribution with Hazen plotting position is given by \( y = 125.5e^{0.767x} (R^2 = 0.975) \), Weibull plotting position gives \( y=128.3e^{0.728x} (R^2=0.986) \) while California plotting position is \( y = 125.6e^{0.789x} (R^2 = 0.957) \). The Root Mean Square Error (RMSE) for Hazen, Weibull and California plotting positions are 6.011, 7.322 and 14.322 respectively. The return periods for 25, 50 and 100 year period with Hazen plotting position are 119.24, 113.14 and 110.10 with absolute differences of 4.7467, 6.2603 and 7.0170 respectively. The return periods for 25, 50 and 100 year period with Weibull plotting position are 121.74, 117.09 and 114.77 with absolute differences of 2.3516, 0.5763 and 0.3113 respectively. Also, the return periods for 25, 50 and 100 year period with California plotting position are 122.02, 117.53 and 114.76 with absolute differences of 2.9466, 1.1730 and 0.2338 respectively.

3.1 Discussions

From the distributions and plotting position charts (Figures 3-5), it was observed that all the probability distributions matched with Weibull plotting position gave similar values near the center of the distribution but varied considerably in the tails as is shown in Figures (3c, 4c, and 5c). This observation was in agreement with Hann (1994). Furthermore, the Weibull plotting position when matched with Normal, Log-normal and Log Pearson Type III distributions have the highest coefficient of determinations of 0.967, 0.987, and 0.986 respectively.

![Figure 3a Normal distribution with Hazen plotting position](image3a)

![Figure 3b Normal distribution with Weibull plotting position](image3b)
The three plotting positions and probability distributions shows that Hazen plotting position gave minimal errors with the root mean square errors of 6.988, 6.390, and 6.011 for Normal, Log-normal, and Log-Pearson Type III probability distributions respectively (Table 2). This is an indication to the fact that using Hazen plotting position to predict statistically, the central tendency of the predicted values to deviate from the
observed flows will be minimal for the period under consideration.

The minimum absolute differences of 2.3516 and 0.5763 at return periods of 25-year and 50-year were obtained under the Log-Pearson Type III distribution when matched with Weibull plotting position respectively, while an absolute difference of 0.2338 at return periods of 100-year was obtained under the Log-Pearson Type III distribution when matched with California plotting position (Table 2). This implies that the magnitudes of error inherent between observed and predicted maximum flows at these different return periods are relatively minimal. Comparing the probability distributions, it was observed that the Log-normal distribution, there was a considerable reduction in the magnitude of the absolute differences for all the plotting positions. However, Log-Pearson Type III distribution with the least absolute differences for all the plotting positions is the best distribution among the three for Ona river under Ogun-osun river basin study location.

The findings above were in contrast with the works of Adeboye and Alatise (2007) which reported that Normal distribution combined with Weibull formula gave the best fit. These deviations may be as a result of differences in the definitions of frequency factor and skewness coefficient in Normal distribution and Log Pearson Type III Distribution respectively. However, similar results were obtained in this work when compared with Adeboye and Alatise (2007) when California plotting position were matched with Log-Pearson Type III for the river basin in the rain forest belt zone of Nigeria.

The comparison of predicted flows with 25, 50, and 100 years return period for different probability distributions (Figure 6) showed that normal distribution deviated greatly from the Log-normal and Log-Pearson Type III distributions for the different plotting positions under consideration. Observed deviations of the normal distribution may be attributed to the failure of not transforming the data on a logarithmic scale which was reported by GREHYS (1996) as a must for obtaining an estimate of the mean flood on an un-gauged catchment. Hence there is the need to consider other distribution characteristics other than the graphical appearance.

<table>
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<tr>
<th>Plotting Positions</th>
<th>Probability Distributions</th>
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<th>Log-Normal</th>
<th>Log-Pearson Type III</th>
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4 Conclusions

The following conclusions were drawn from the study:
- The annual maximum discharges for Ona River at Ogun-Oshun River Basin vary in magnitude ranging from 121.19 m$^3$/s to 271.01 m$^3$/s within between 1982 and 1999.
- All the three distributions had the highest coefficient of determination using Weibull’s plotting position.
- Also, all the distributions had minimum (RMSE) when matched with Hazen plotting positions.
- Accumulation of the absolute differences between observed and predicted flows, California gave the least value of 209.46 followed by Weibull and Hazen with values of 210.99 and 241.77 respectively. Generally, California and Weibull plotting positions predicted observed flows better than Hazen plotting position.
- Hence, in predicting a maximum flood with a return period of 25-year and 50-year period, the Log-Pearson Type III probability distribution should be used with Weibull plotting position while the use of California plotting position with Log-Pearson Type III probability distribution is suggested for the prediction of 100-year maximum flood return period for the Ona river basin in the rain forest belt zone of Nigeria.

References


## Appendix

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<th>Max Observed Q/Ft³·s⁻¹</th>
<th>Max Observed Q/m³·s⁻¹</th>
<th>Rank</th>
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### Normal Distribution

- Predicted Q (Hazen): 190.87, 191.97, 192.1
- Predicted Q (Weibull): 193.32, 194.29, 194.55
- Predicted Q (California): 195.77, 196.61, 197

### LogNormal Distribution

- Predicted Q (Hazen): 117.36, 125.85, 126.65
- Predicted Q (Weibull): 142.86, 145.67, 146.91
- Predicted Q (California): 142.74, 145.7, 147

### Log Pearson Type III Distribution

- Predicted Q (Hazen): 115.52, 124.66, 125.52
- Predicted Q (Weibull): 132.43, 136.89, 137.98
- Predicted Q (California): 150.91, 153.05, 154.56