Assessment of the flow properties of crushed grain products depending on the granulometric condition

Christian Fürll, Thomas Hoffmann*

(Leibniz Institute for Agricultural Engineering Potsdam-Bornim, Max-Eyth-Allee 100, 14469 Potsdam, Germany)

Abstract: Several methods were developed to assess the flowability of bulk material. Jenike introduced flowability values to assess the flowability of bulk material like mineral raw material under given loads. A method developed by Carr uses several granulometric parameters to describe the flow properties without influence on the load. The aim of the own study was to evaluate the recommendations of Jenike and Carr regarding to their application onto crushed grain. Therefore measurements with grainy and powdery grain products were carried out to determine flow properties regarding Jenike and Carr. Additionally unloading tests with a level model silo were performed to find out the influence of the outlet diameter. The evaluations of the flow properties according to Jenike and Carr deliver partially different textual evaluations. The flowability to Jenike and the flow index according to Carr drop abruptly in the range \(Q_3(0.2 \text{ mm}) = 5\% ... 20\%\). This shows that the flow properties are mainly influenced by their share of cohesive fines. A classification of the flow properties according to Jenike is more realistic for crushed grain.

Keywords: crushed grain, flow properties, Jenike shear cell, Carr index

Introduction

The bases of calculation for silo designs conducive to flow and for measuring technology to determine flow properties have been described in relevant literature for about 50 years now (Schwedes, 1968; Schwedes, 1980). Up to now, several test devices and methods were developed for the assessment of parameters attributed to the flowability (Schwedes, 2003; Schulze, 2008). The parameters of interest depend on the given problems.

Key parameters obtained by measuring the flow properties with shear testing equipment and used for designing silos are the internal friction, the uniaxial strength of the material, the wall friction, the stress-dependent bulk or storage density and the flowability. Measuring equipment used worldwide includes the translation shear cell according to Jenike (Schwedes, 1968) and the ring shear tester according to Walker (1966), or in an essentially more modern design according to Schulze (1994). The flow properties are still chiefly assessed according to a classification by Jenike (Schwedes, 1968) on the basis of the flowability \(ff\), which is calculated from the division of consolidation stress \(\sigma_1\) and uniaxial compressive strength \(\sigma_c\):

\[
ff = \frac{\sigma_1}{\sigma_c}
\]  

However, the influence of density on the flow properties is not taken into account. Probably for this reason Jenike remarks in a publication of 1975 (Jenike, 1975) that his original recommendation was too simple. He therefore suggests using the smallest width of a rectangular unloading outlet to assess the flowability. In this way the material density is also taken into account.

If samples to be compared differ strongly in their bulk density, Schulze (2008) proposes the calculation of an adjusted value for the flowability \(ff_p\) with
consideration of the bulk density $\rho_b$ and the density of water $\rho_w = 1000 \text{ kg/m}^3$ (liquid water at 0°C, one bar):

$$ff_\rho = ff_c \cdot \frac{\rho_b}{\rho_w} \quad (2)$$

The flowability $ff_\rho$ is not physically based, but Equation (2) gives a non-dimensional value which is called the “density-weighed flowability”.

The density influence is taken into account when assessing the flow properties according to a proposal by Peschl (1989) and Shear-Test GmbH (ShearTest, 2015). This defines a relative flowability $FLR$, calculated in accordance with the following equations:

$$FLR = \frac{\sigma_1 - \sigma_2}{\sigma_c} \quad (3)$$

Where, $\sigma_1$ and $\sigma_2$ are the major principal stresses of the Mohr cycle which is tangent on the end point of the yield locus, and $\sigma_c$ is the uniaxial compressive strength. The density influence is included in the function of the relative flowability $FLA$:

$$FLA = FLR \cdot \frac{\rho}{\rho_w} \quad (4)$$

Where, $\rho/\rho_w$ is the ratio of the bulk density of the material examined to the density of water.

In examinations of mineral raw materials, for instance by Kurz and Münz (1975), Münz (1976) and Höhne and Schünemann (1980), connections between particle size and the parameters of various particle size functions and the flow properties were determined. They were also ascertained for crushed grain products (Fürll, 1978; Fürll, 2007) and various agricultural model products (Fürll, 1978; Fürll, 1979). Cohesionless properties are only available for crushed grain as of particle sizes from 200 to 300 µm (Fürll, 2006; Fürll, 2007). This agrees with the results of Nothdurft (1977), who states from experiences with a wide range of agricultural bulk materials that as of particle sizes $x > 250$ µm, the bulk materials are generally cohesionless. Ogniwek (1979) derives from test results with nine different product samples, including wheat, wheat flour and crushed soy, that substances are cohesive already as of a mass component $Q_3(200 \mu m) \geq 3\%$. This was also confirmed by our own test results (Fürll, 2006; Fürll, 2007).

Results obtained by Höhne and Schünemann (1980) document that the flow properties are a function of density. They ascertained that with mineral raw materials, particle sizes of $x > 5$ µm already possess cohesionless properties.

The granulometric condition sizes are also included directly and indirectly in the evaluation of flow properties by Carr (1965). He awards points for the angle of repose, spatula angle, coefficient of compressibility and the “cohesion according to Carr” or the coefficient of uniformity of the particle size distribution of a material. The flowability is classified according to a given scale from 0 (absolutely not flowing) to 100 (absolutely free flowing). The advantage of this method lies in the generally easy determinability of the parameters. A disadvantage is that the measurements are carried out without influence on the load and that the result obtained does not represent any possibility of calculating container geometries.

Altogether it must be ascertained that the influence of the particle density is not taken into account on assessing the flow properties. This leads to misinterpretations, above all when assessing the flow properties from results of shear tests with the aid of the flowability according to Jenike ffc and the classification he recommended.

Several results about the flow behaviour of mineral materials or materials from the pharmaceutical industry were published. But only few publications deal with agricultural materials like whole wheat grains (Chang, 1988), distillers dried grains with solubles (Ganesan et al., 2008), cereal powders and non-starch powders (Stasiak and Molenda, 2008), starch of maize, wheat and potatoes (Stasiak et al., 2013), or shredded grain products (Fürll and Hoffmann, 2013).
The aim of the own investigation was to determine the flow properties of grainy and powdery grain products in accordance with the early recommendations of Jenike and using the methods according to Carr. The results and their practical relevance are then to be checked by experimental investigations. A correction of the recommendations provided in literature to date is then to be derived from the results, taking the influence of the material density into account. In addition, the study examines whether there is any connection between the flowability $f_{fc}$ and the geometry of the unloading outlet.

2 Materials and methods

2.1 Flowability according to Jenike

The flowability according to Jenike $f_{fc}$ is calculated from the ratio of the principle stress $\sigma_1$ (consolidation stress) of the Mohr’s circle which is tangent to the end point of a yield locus to the uniaxial compressive strength $\sigma_c$ which is the maximum principle stress of the Mohr’s circle which touches on the yield locus and which second principle stress is zero (Equation (1)).

According to its value, the flow properties of a bulk good are classified on the basis of an original recommendation by Jenike:

- $f_{fc} < 2$ very cohesive, non-flowing
- $2 < f_{fc} < 4$ cohesive
- $4 < f_{fc} < 10$ easily flowing
- $10 < f_{fc}$ free flowing

As Jenike later suggested using the minimal necessary outlet width as dimension for the flowability ($f_{fc}$), it is interesting whether there is a connection between outlet diameter $d_A$ and flowability $f_{fc}$. If $\sigma_{c \text{krit}}$ is replaced by $\sigma_1/f_{fc}$ in the equation for calculating the outlet widths (Equation (5)), it becomes apparent that $d_A$ is inversely proportional to $f_{fc}$ (Equation (6)).

$$b, d_A = H(\Theta) \cdot \frac{\sigma_{c \text{krit}}}{\rho_b \cdot g} \quad (5)$$

$$b, d_A = H(\Theta) \cdot \frac{\sigma_1}{f_{fc} \cdot \rho_b \cdot g} \quad (6)$$

$\Theta$ function that takes the geometrical form of the outlet into account, 
$\sigma_{c \text{krit}}$ critical uniaxial compressive strength, N/m²
$\rho_b$ bulk density during storage, kg/m³
$g$ acceleration due to gravity, m/s²

In this equation the principle stress $\sigma_1$ can be assumed to be virtually constant, while all $f_{fc}$-values have been obtained as mean value from the experimentally determined yield loci.

Furthermore, the influence of density becomes clear from Equation (6).

2.2 Flow index according to Carr

Carr (1965) determines the flow index $f_{ffc}$ according to a given evaluation scale from the measured values of angle of repose, spatula angle, compressibility and cohesion or coefficient of uniformity. By the method of Carr the values for the four criteria mentioned are determined by separate tests and are assigned to a list of score points. Each value can lie between 0 points (bad) and 25 points (excellent). The flow index according to Carr $f_{ffc}$ is the sum of all four score points and can reach in the best case 100 points. The experimental method is specified in a work instruction (ASTM D6393).

To determine the angle of repose the material is poured by a vibrating funnel on a round plate. The angle of repose is measured. Due to determine the angle spatula, a spatula is lifted from the bottom of a container with the material and the slope angle of the material on the spatula is measured. The compressibility $C$ is calculated from the packed bulk density $P$ and the loose bulk density $L$:

$$C = \frac{P-L}{P} \cdot 100 \% \quad (7)$$

The material residue remaining after test screening on three screens is used to determine the cohesion $K$.
according to Carr. First of all 2 g is weighed out as sample mass. The cohesion is obtained in % from the residual mass on the upper screen $m_{RoS}$, the middle screen $m_{RmS}$ and the lower screen $m_{RuS}$:

$$K = \frac{100}{2} \left( m_{RoS} + m_{RmS} \cdot \frac{3}{5} + m_{RuS} \cdot \frac{1}{5} \right)$$

(8)

It is measured if the determination method allows this. If this is not possible, the coefficient of uniformity $U$ from the screen analysis is used:

$$U = \frac{x_{60;3}}{x_{10;3}}$$

(9)

Each value is determined from three repeat tests so that the flow index $f_{fc}$ according to Carr of a bulk material sample results from 15 individual experiment tests.

2.3 Test material

A standard commercial dry compound feed with a barley component of 72% and a rye component of 20% is used as test material. In order to be able to determine the influences of the granulometric parameters particle size and particle size distribution, the samples are made up in accordance with the bi-parametric and tri-parametric logarithmic normal distribution (LND). The granulometric condition of pig finishing feed is varied for the bi-parametric logarithmic normal distribution by systematically altering the central value $x_{50;3}$ and the standard deviation $\sigma_{g}$. For the material systems with tri-parametric logarithmic normal distribution, the standard deviation is $\sigma_{g}^* = \text{constant } \sigma_{g}^* = 0.9$. This corresponds to the real value in practice. The granulometric condition is varied by systematically changing the central value $x_{50;3}^*$ and the maximum particle size $x_{\text{max}}$. At constant standard deviation $\sigma_{g}^*$, the central value $x_{50;3}^*$ and the maximum particle size $x_{\text{max}}$ determine both the half value and the width of the particle size distribution function.

2.4 Performance of the experiments

Shear strength measurements are carried out to determine the flow properties according to Jenike. The ring shear tester and the translation shear tester according to Jenike can be considered measuring equipment here. As the ring shear tester is more suitable for compressible organic substances than the translation shear tester due to the infinite shear path, it was used in this case.

The following parameters are determined from a yield locus family and the wall yield locus: angle of internal friction $\varphi_i$, effective angle of friction $\varphi_e$, stationary angle of friction $\varphi_{St}$, uniaxial compressive strength $\sigma_c$, cohesion $\tau_c$ (shear strength at normal stress $\sigma_N = 0$), wall friction angle $\varphi_x$ and adhesion $\tau_{cx}$ (wall shear strength at the normal stress $\sigma_N = 0$).

The experimental unloading tests were carried out with a level model silo. The dimensions height x width are 100 x 50 cm. The hopper angle $\theta$ can be varied infinitely. The gap width of the unloading outlet can also be varied infinitely up to 100 mm.

3 Results and discussion

3.1 Flowability according to Jenike

Given the same values of the flowability $f_{fc}$, higher values for the unloading outlet are obtained with lower densities (Schulze, 2012). If we use $\sigma_1 = 3$ kPa in Equation (5) for the consolidation stress and $\rho_L = 700 \text{ kg/m}^3$ for the bulk density (grain products), $\rho_L = 1500 \text{ kg/m}^3$ (mineral raw materials) and $\rho_L = 2500 \text{ kg/m}^3$ (iron ores) and for $H(\theta) = 2$ (simplified), then for $d_A = f(f_{fc})$ we obtain the following family of curves with the density as parameter (Figure 1):
Jenike thus subsequently recognised that the flowability is also a function of the density.

The smaller the particle size $x_{50,3}$, the lower the flowability according to Jenike (Figure 2). For particle sizes $x < 0.2$ mm the flowability reaches very low values of $f_{c} < 10$.

This is evident from examinations with uniform grain ($0.08 < \sigma_{\zeta} < 0.22$) and coincides with the results of measurements conducted in parallel on the tensile strength of the angle of repose, spatula angle and compressibility (Fürll and Hoffmann, 2013).

In material systems with a particle size distribution according to the bi-parametric LND, the influence of the standard deviation on the flow function in the range examined is inversely proportional. The central value $x_{50,3}$ influences the flow properties according to the third power (Figure 3).

Figure 1  Connection between the outlet diameter $d_{A}$ and flowability $f_{c}$ in accordance with the dimensioning equation for unloading outlets at a consolidation stress $\sigma_{1} = 3$ kPa

Figure 2  Flowability $f_{c}$ according to Jenike as a function of the particle size $x_{50,3}$ of grain – dry compound feed

- Grain, 700 kg/m³
- Mineral raw material, 1500 kg/m³
- Iron ore, 2500 kg/m³

\[ f_{c} = 181.0 (x_{50,3})^{2.23} + 4.0 \]
\[ R^2 = 0.701 \]

Dry matter content
DM = 88.4 %
Standard deviation of the lognormal distribution
$0.08 < \sigma_{\zeta} < 0.22$
The share of cohesive fines results in a sudden drop in the range $Q_3(0.2 \text{ mm}) = 5\%$ to $20\%$, for both material systems with a particle size distribution in accordance with the bi-parametric LND and systems in accordance with the tri-parametric LND (Figure 4).

The verbal textual assessment of the flow properties results in a few differences between the classifications by Carr and Jenike (Table 1). In the area that Jenike estimates as “easily flowing”, Carr states “not good” or even “poor”.

### 3.2 Flow index according to Carr

For the grain/dry compound feed examined, in principle the same trends apply for the flow index according to Carr $f_{cc}$ as a function of the granulometric condition as for the results of the flow function according to Jenike $f_c$. Material systems with a particle size distribution according to the bi-parametric LND display a dependence of the flow index $f_{cc}$ on the ratio $x_{50:3}/\sigma_c$ (Figure 5).

The course of the flow index according to Carr as a function of the cohesive fines share $Q_3(x_{FG})$ brings about a sudden drop in the values in the range of $Q_3(0.2 \text{ mm}) = 50\%$ to $20\%$, as in the flow function according to Jenike $f_c$.

### 3.3 Outflow from a model container
The examinations to determine the minimum gap width \(b\) as a function of the hopper inclination for disturbance-free outflow were carried out with uniform grain material systems and with material systems with a particle size distribution according to the tri-parametric logarithmic normal distribution. At the minimum gap width for cohesionless materials stated by Kvapil (1959), uniform grain substance systems only flow undisturbed as of a particle size \(x > 1.2\) mm and a hopper inclination angle \(\Theta = 15^\circ\) (Figure 6). At a linear connection between the minimum gap width and particle size, completely cohesionless behaviour only exists as of a particle size \(x > 0.8\) mm. For particle sizes \(x < 0.2\) mm, the minimum gap widths are rise abruptly. This corresponds to a flowability value \(ff_c\) according to Jenike of \(ff_c < 10.0\).

Material systems with a real particle size distribution, expressed by the Sauter diameter \(d_{St}\), require a larger minimum gap width \(b\) than uniform grain material systems in the range \(d_{St} > 0.5\) mm to be

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Central value (x_{50.3}) mm</th>
<th>Standard deviation (\sigma_c)</th>
<th>Flow index (ff_{cc}) acc. to Carr</th>
<th>Assessment of flowability (ff_{cc}) acc. to Carr</th>
<th>Flowability (ff_c) acc. to Jenike</th>
<th>Assessment of flowability (ff_c) acc. to Jenike</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.550</td>
<td>0.49</td>
<td>58.25</td>
<td>not good</td>
<td>14.6</td>
<td>free flowing</td>
</tr>
<tr>
<td>2</td>
<td>0.540</td>
<td>0.28</td>
<td>69.00</td>
<td>normal</td>
<td>28.1</td>
<td>free flowing</td>
</tr>
<tr>
<td>3</td>
<td>0.510</td>
<td>0.11</td>
<td>79.00</td>
<td>good</td>
<td>37.5</td>
<td>free flowing</td>
</tr>
<tr>
<td>4</td>
<td>0.165</td>
<td>0.51</td>
<td>44.75</td>
<td>not good</td>
<td>6.2</td>
<td>easily flowing</td>
</tr>
<tr>
<td>5</td>
<td>0.165</td>
<td>0.30</td>
<td>51.00</td>
<td>not good</td>
<td>6.6</td>
<td>easily flowing</td>
</tr>
<tr>
<td>6</td>
<td>0.154</td>
<td>0.11</td>
<td>57.50</td>
<td>not good</td>
<td>14.0</td>
<td>free flowing</td>
</tr>
<tr>
<td>7</td>
<td>0.075</td>
<td>0.49</td>
<td>37.50</td>
<td>poor</td>
<td>4.4</td>
<td>easily flowing</td>
</tr>
<tr>
<td>8</td>
<td>0.076</td>
<td>0.29</td>
<td>42.00</td>
<td>not good</td>
<td>4.6</td>
<td>easily flowing</td>
</tr>
<tr>
<td>9</td>
<td>0.072</td>
<td>0.10</td>
<td>46.00</td>
<td>not good</td>
<td>4.1</td>
<td>easily flowing</td>
</tr>
<tr>
<td>10</td>
<td>0.720</td>
<td>0.22</td>
<td>80.00</td>
<td>completely good</td>
<td>79.7</td>
<td>free flowing</td>
</tr>
<tr>
<td>11</td>
<td>0.720</td>
<td>0.22</td>
<td>80.00</td>
<td>completely good</td>
<td>39.2</td>
<td>free flowing</td>
</tr>
</tbody>
</table>

Figure 6  Minimum gap width \(b\) for disturbance-free outflow of pig finishing feed from a level laboratory container as a function of the parameters particle size \(x\) of uniform grain material systems and the hopper inclination angle.
Hopper inclination angles $\theta \leq 45^\circ$ show a similar relation between Sauter diameter $d_{St}$ and minimum gap width and can be combined (Figure 7). They lead to mass flow. Greater angles $\theta \geq 60^\circ$ result in core flow.

A Sauter diameter $d_{St} = 0.5$ mm requires in the case of mass flow ($\theta \leq 45^\circ$) a minimum gap width of 20 mm. For the practical application of mass flow, a slightly greater gap width of 25 mm should be selected. Therefore the minimum gap width $b$ (25 mm) is 50 times greater than $d_{ST}$ (0.5 mm). For core flow a Sauter diameter $d_{St} = 0.5$ mm demands a gap width greater 40 mm. Considering a safety margin, $b > 50$ mm should be chosen. Therefore, the following relations can be drawn up for the minimum gap width from the test results for disturbance-free outflow:

- $b > 50 \ d_{St}$ for mass flow
- $b > 100 \ d_{St}$ for core flow

It is not possible to state the minimum gap width as a function of the Sauter diameter in the cohesive range. It is apparent from the experiments and on the basis of the flowability according to Jenike and the flow index according to Carr that even slight components of cohesive fines $Q_3(0.2 \text{mm}) > 7.0\%$ substantially reduce the flow properties, above all for outflow in the core flow (Figure 7).

### 3.4 Comparison of the results

It becomes clear from the results of the experimental studies that the textual evaluation of the flow properties produces some differences between the classification by Carr and the original recommendation by Jenike (Table 1). According to the experiments conducted to determine the minimum gap width, it can be appraised that the assessment according to Carr is more realistic for crushed grain products.

Despite this assessment, the method by Carr is more suitable for verbal assessments of the flowability (Schwedes, 2003; Ganesan et al., 2008). In contrast, the shear test according to Jenike delivers physical based...
material parameters. These parameters are the basis for dimensioning the silo outlets.

Up to the time of publishing his classification, Jenike probably only conducted examinations with mineral raw materials that on average possess a three to four times higher bulk density than grain products. As the storage density according to Jenike is used inversely proportionately to calculate the outlet geometry, the differences can be explained (Equation (4)).

That is why the following classification is suggested on the basis of our own examinations of crushed grain products for these substances:

- \( f_{fc} < 6 \) very cohesive, non-flowing
- \( 6 < f_{fc} < 30 \) cohesive
- \( 30 < f_{fc} < 50 \) easily flowing
- \( 50 < f_{fc} \) free flowing

A hyperbolic curve is obtained from the \( f_{fc} \)-values for the flowability determined from shear tests and the calculated outlet diameters (Figure 8). This function curve coincides approximately with the values calculated in accordance with Figure 1.

![Figure 8](image)

**Figure 8** Connection between the flowability \( f_{fc} \) and the calculated diameter \( d_A \) of the unloading outlet from experiments with crushed grain

4 Conclusions

At particle size distributions according to the bi-parametric LND, the ratio of central value \( x_{50.3} \) / standard deviation \( \sigma_c \) influences the flowability \( f_{fc} \) according to Jenike and the flow index \( f_{fc}c \) according to Carr inversely proportionately according to statistically secured connections. The flow properties of particle size distributions according to the tri-parametric LND at constant transformed standard deviation \( \sigma_c^* \) are above all a function of the transformed central value \( x_{50.3}^* \). The causes lie in the outstanding influence of the cohesive fines \( Q_3(x_{FG}) \). The flowability \( f_{fc} \) according to Jenike and the flow index \( f_{fc}c \) according to Carr drop abruptly in the range of \( Q_3(x_{FG}) = 5\% \) to \( Q_3(x_{FG}) = 25\% \). This connection also becomes clear from the results of experimental unloading tests and confirms that the flow properties are determined above all by their share of cohesive fines. For grain/dry compound feed, the boundary particle size below which the cohesive properties increase progressively is \( x_{FG} = 0.2 \) mm.

In the textual evaluation of the flow properties roughly the same tendencies are obtained using the methods according to Jenike and Carr, but partially very different textual evaluations result. This is attributable to the fact that in his original classification Jenike did not take the material density into account. He corrected this later with the recommendation that the outlet widths be used as measure for the flowability. On the basis of the examinations conducted a more realistic classification is suggested for evaluating the flow properties of crushed grain products with the help of the flowability \( f_{fc} \) according to Jenike.

However, it is better to use the required opening width as a value for characterising the flowability, as proposed by Jenike (1975).

References


